MATH 464 (THEORY OF PROBABILITY) HOMEWORK 6

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(1) Suppose $X_1, \dots, X_n$ are independent random variables with $\mathbb{E}(X_i) = \mu$ and $\mathrm{Var}(X_i) = \sigma$ for all $i = 1, \dots, n$, Find $\mathbb{E}((X_1 + \dots + X_n)^2)$.

(2) Let X and Y be independent discrete random variables. Given that

$$\mathbb{E}(X^n) = 2^{n-1}, \text{ and } \mathbb{E}(Y^n) = \begin{cases} 0, & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases} \quad \text{for } n = 1, 2, \dots$$

(a) Give an example of such random variables X and Y.
(b) Let $Z = 2X + Y$. Find the mean and variance of Z.
(c) Let $W = Y^2 - 2YX^2$. Find the mean and variance of W.

(3) Let X and Y be independent discrete random variables with $\mathrm{Var}(X) = 4$ and $\mathrm{Var}(Y) = 3$. Find $\mathrm{Var}(3X - 2Y + 5)$.

(4) Suppose that U and V are two independent random variables, each take the values of -1 and 1 only, and $\mathbb{P}(U = 1) = \mathbb{P}(V = 1) = 1/2$. A third random variable W is defined by $W = UV$. Show that the random variables U, V, and W are pairwise-independent but they are not independent.

(5) Consider the following experiment: Roll two fair, four sided dice. Consider the following discrete random variables:
X = the number of odd dice.
Y = the number of even dice.
Clearly, each of X and Y have range $\{0, 1, 2\}$.
(a) Find $f_{X,Y}(x, y)$. Give your answer in tabular form.
(b) Determine whether or not X and Y are independent.
(c)) Find $E(XY)$.

(6) Suppose that X and Y are discrete random variables and that you know the joint probability mass function of X and Y is:

$$f_{X,Y}(x, y) = \alpha^{x+y+1} \quad \text{for } x, y = 0, 1, 2 \quad \text{with some } \alpha > 0.$$

Find $E(XY)$ and $E(Y)$.

(7) Let $X_1, X_2, \cdots, X_{100}$ be independent discrete random variables. Suppose that each of them is a Poisson random variable with $\lambda = 2$. Consider

$$\overline{X} = \frac{1}{100} \sum_{j=1}^{100} X_j$$

which is sometimes called the *sample mean*. Find the mean and variance of $\overline{X}$.